Low Temperature Phase of Asymmetric Spin Glass Model in Two Dimensions

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Abstract

We investigate low temperature properties of a random Ising model with $+J$ and $-aJ(a \neq 1)$ bonds in two dimensions using a cluster heat bath method. It is found that the Binder parameters g_L for different sizes of the lattice come together at almost the same temperature implying the occurrence of the spin glass(SG) phase transition. From results of finite size scaling analyses, we suggest that the SG phase really occurs at low temperatures which is characterized by a power law decay of spin correlations.

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Spin glasses have attracted great challenge for computational physics in these two decades. It is widely believed now in the bond-random Ising model that spin glass (SG) transitions occur at a finite, non-zero temperature $T_c \neq 0$ in three dimensions(3D) [1–4] and at zero temperature $T_c = 0$ in two dimensions(2D) [4–8]. Recently, the present authors [9] reexamined the SG phase transition of the $\pm J$ Ising model on a square lattice of $L \times L$ by means of an exchange Monte Carlo method [10] and found that the Binder parameters g_L for $L \leq 16$ intersect at $T \neq 0$. They also found that better finite-size scaling(FSS) fits of the spin glass susceptibility χ_{SG} are obtained when $T_c \neq 0$. These results imply the occurrence of the SG phase transition at $T_c \neq 0$. If so, it is quite interesting, because it disproves the belief of $T_c = 0$. However, there remain two problems which should be considered to suggest $T_c \neq 0$ in 2D. One is that g_L for a smaller lattice almost saturates below a rather high temperature [7] and its saturation value slightly increases with L [9]. Therefore, it is difficult to see whether the intersection of g_L for smaller L suggests the presence of the SG phase at $T_c \neq 0$ or is merely due to a finite size effect. The other is that it is still open whether or not the model really exhibits the nature of the SG phase at $T < T_c$, because the estimated transition temperature T_c is a slightly lower than the lowest temperature which is reached in the simulation. The problems would be solved, if we study the same model on bigger lattices at lower temperatures. The saturation of g_L at rather high temperatures, however, may be removed, if we treat an asymmetric random Ising model with $+J$ and $-aJ(a \neq 1)$ bonds, because the energy gap of $2|1-a|J$ in that model between the ground state and the lowest excitation state is much smaller than that of $4J$ in the $\pm J$ model [11], and, if the lattice is rather small, we may study equilibrium properties at any temperature using a cluster heat bath(CHB) method [12,13].

In this Letter, we investigate low temperature properties of the asymmetric random Ising model on the square lattice of $L \times L (L \leq 18)$ using the CHB method. In fact, g_L does not saturate down to a very low temperature. We find that, as the temperature decreases, g_L's for different L meet at almost the same temperature and then increase together. This property rather resembles that of the $\pm J$ model in 3D in which the SG phase transition

occurs at $T_c \neq 0$. We make the FSS analyses and find that g_L and χ_{SG} for different L scale well using a finite, non-zero value of T_c and that the distribution functions $P_L(Q)$ of the spin overlap Q scale at all temperatures below T_c. Thus we suggest that in this model the SG phase occurs at low temperatures which is characterized by a power law decay of the spin correlations. The properties for $T > T_c$ found here are very similar to those of the $\pm J$ model in 2D [9]. We believe, hence, that the SG phase transition also occurs at $T_c \neq 0$ in the $\pm J$ model in 2D.

We start with an Ising model on a square lattice $L \times L$ described by the Hamiltonian

$$H = - \sum_{<i,j>} J_{ij} \sigma_i \sigma_j, \tag{1}$$

where $\sigma_i (= \pm 1)$ are Ising spins and $< ij >$ runs all nearest neighbor pairs. Distributions of bonds for different samples are given as

$$P(J_{ij}) = \frac{1}{2}[\delta(J_{ij} - J) + \delta(J_{ij} + aJ)]. \tag{2}$$

If the lattice is rather small and a free boundary condition is used at least for one direction, we may readily obtain equilibrium spin configurations at any temperature T by using the CHB method, because the cluster defined in Ref. 13 can be chosen as the lattice itself. That is, the cluster is composed of L layers with L spins and exchange fields from the outside are absent. We briefly note the method [14]. For every sample, the weight functions $F_l(\{\sigma_i^{(l)}\})$ can be uniquely determined by using Eq.(7) in Ref. 13, because $h_i^{(l)} = 0$. Once the set of these functions $\{F_l(\{\sigma_i^{(l)}\})\}$ is obtained, the spin configurations of individual layers can be determined successively from the layer (L) to the layer (1) by using Eq.(8) or Eq.(11) in Ref. 13 and random numbers. Thus one of the spin configurations of the lattice is generated. Repeating this procedure, we may generate any number of the spin configurations with the aid of $\{F_l(\{\sigma_i^{(l)}\})\}$. These spin configurations are independent with each other and in accordance with the Boltzmann's weight [13]. For each of the samples, about $M = 200$ spin configurations are generated [15]. We calculate, as well as usual magnetic quantities, an overlap function of the spins between the different spin configurations, $P_L^{(k)}(Q)$, for every sample:

$$P_L^{(k)}(Q) = \frac{2}{M(M-1)} \sum_{n}^{M} \sum_{m(>n)}^{M} \delta(Q - Q_{nm}^{(k)}) \tag{3}$$

with $Q_{nm}^{(k)} = (1/N)\sum_{i=1}^{N} \sigma_i^{(n,k)}\sigma_i^{(m,k)}$, where $\sigma_i^{(n,k)}$ is the ith spin of the nth spin configuration for the kth sample. The overlap function $P_L(Q)$ of the system is given as $P_L(Q) = (1/N_s)\sum_{k=1}^{N_s} P_L^{(k)}(Q)$, where N_s is the number of the samples. Once $P_L(Q)$ is determined, we may obtain various SG quantities. The nth moment of the spin overlap is defined as:

$$[< Q^n >] = \int_{-1}^{+1} Q^n P_L(Q) dQ, \tag{4}$$

where $< \cdots >$ and $[\cdots]$ mean the spin configuration(thermal) average and the sample average, respectively. The SG susceptibility χ_{SG} is determined from

$$\chi_{SG} = N[< Q^2 >], \tag{5}$$

and the Binder parameter g_L from

$$g_L = (3 - [< Q^4 >]/[< Q^2 >]^2)/2. \tag{6}$$

We have performed this CHB simulation of the model of Eq.(1) with $a = 0.8$ for $L \le 18$. The numbers of the samples are $N_s = 4000$ for $L \le 14$ and $N_s = 1000$ for $L = 16$ and 18.

Figure 1 shows $P_L(Q)$ for different L. For every size L, the shape is symmetric with respect to $Q = 0$ and the peaks at $Q \sim \pm 1$ become steeper as the temperature decreases. Figure 2 shows plots of g_L against T. As the temperature decreases, g_L's for different L meet at almost the same temperature of $T \sim 0.2J$ and then increase together. This behavior is quite similar to that of g_L of the $\pm J$ model in 3D in which the SG phase transition occurs at $T_c \ne 0$ [1].

We examine the results in more detail using the FSS analyses. First, we perform the FSS plots of g_L and χ_{SG} to estimate the value of T_c. If the SG transition occurs at T_c, g_L and χ_{SG} scale as

$$g_L = G(\epsilon L^{1/\nu}), \tag{7}$$

$$\chi_{SG} = L^{2-\eta} X(\epsilon L^{1/\nu}), \tag{8}$$

where $\epsilon = (T - T_c)/J$, ν is the exponent of the correlation length, η is the exponent which describes the decay of the spin correlation at $T = T_c$, and G and X are some scaling functions. Having assumed $T_c \simeq 0.19J$, we could obtain good scaling plots for $T \geq T_c$. Typical examples are shown in Figs. 3(a) and 3(b). Of course, the values of ν estimated from both the scaling plots are almost the same. We also examined the other possibility of $T_c = 0$. As for χ_{SG}, we could scale the data only in the neighborhood of $T = 0$ using $\eta \sim 0$ and $\nu \sim 2.6$. As for g_L, on the other hand, we could never scale the data using any plausible value of ν. The scaling plots for $T_c = 0$ are shown in Figs. 4(a) and 4(b). These results clearly reveal that, if a conventional phase transition occurs, the transition temperature is $T_c \sim 0.19J$, not $T_c \sim 0$. However, the data for $T < T_c$ deviate from the scaling plots.

Next we examine $P_L(Q)$ itself to see whether or not the SG phase is realized below $T < T_c$. If the phase transition occurs at $T = T_c$, $P_L(Q)$'s for different L will scale as

$$P_L(q) = L^{\eta/2} P(qL^{\eta/2}) \quad \text{at} \quad T = T_c. \tag{9}$$

Since $P_L(Q)$ for smaller L has a considerable weight at $Q = \pm 1$ and, for $L \leq 16$, the peak height rather decreases with increasing L, we could not scale all the data over the entire range of Q. These difficulties may, however, come from finite size effects. In fact, the weights at $Q = \pm 1$ become smaller as L increases and, for $T \leq 0.2J$, the peak height for $L = 18$ becomes larger than that for $L = 16$. If we overlook the discrepancy around the peak of $P_L(Q)$, the data scale for $L \geq 10$ at $T \sim T_c$ by using the same value of $\eta \sim 0.14$ for χ_{SG}, which is shown in Fig. 5(a). However, the data for $T < T_c$ can also be scaled by using a smaller value of η as shown in Fig. 5(b) [16]. A natural interpretation of this result is that the model is close to criticality at all temperatures below T_c like the XY ferromagnet in 2D [17]. This picture is, of course, compatible with the fact that g_L and χ_{SG} scale only for $T \geq T_c$. We suggest, hence, that the SG phase really occurs below $T_c \sim 0.19J$ which is characterized by a power law decay of the spin correlations.

Our present result is in agreement with our previous result of $T_c \neq 0$ in the $\pm J$ model [9]. Especially, the value of $\nu \sim 0.18$ in the $\pm J$ model is in good agreement with that obtained

in the present model [18]. Thus we predict that the occurrence of the SG phase at $T_c \neq 0$ is the common nature of 2D random Ising models with a discrete distribution of bonds [19]. Our prediction of $T_c \neq 0$ appears incompatible with the previous belief of $T_c = 0$. We think, however, that these are not necessarily incompatible, because the previous authors have only concluded that *their data are not incompatible with the prediction that $T_c = 0$, and have not ruled out the possibility of such low T_c as estimated here.* The thing that was certainly suggested by the previous studies is that, at $T = 0$, no long-range order exists and the spin correlation decays according to the power law [5,6,8]. This is compatible with our result with $T_c \neq 0$. Of course, further studies are necessary to confirm the prediction of $T_c \neq 0$. We believe that the present result will stimulate not only the computational physics but also experimental studies, because the bond distributions of real SG materials will be asymmetric.

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[14] Note that, as shown by Morgenstern and Binder [5], the partition function of the lattice can be exactly obtained from $Z_L = \sum_{\{\sigma_i^{(L)}=\pm 1\}} F_L(\{\sigma_i^{(L)}\})$. But it is not easy to extract SG quantities from Z_L itself(e.g., N. Kawashima, N. Hatano, and M. Suzuki, J. Phys A **25**, 4985 (1992)). The CHB method is a Monte Carlo method which gives spin configurations of the lattice. Thus the spin overlap function $P_L(Q)$ at low temperatures is readily obtained.

[15] We have examined the accuracy of results comparing data averaged over M spin config-

urations with those over $M/2$ spin configurations and found that the difference is less than 0.1% after sample averages are taken out.

[16] These values of η are consistent with those estimated from the slope of the log-log plots of χ_{SG} versus L.

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[18] The value of $\eta \sim 0.20$ in the $\pm J$ model is somewhat different from $\eta \sim 0.14$ in the present model. But the value of η is suggested to change with the form of the distribution of bonds(e. g. L.Bernardi and I. A. Campbell, Phys. Rev. B **52**, 12501 (1995)).

[19] At present, we could not give the same prediction for the model with a Gaussian distribution of bonds, because the distribution of excitation energies of that model is quite different from that of the present model.

FIGURES

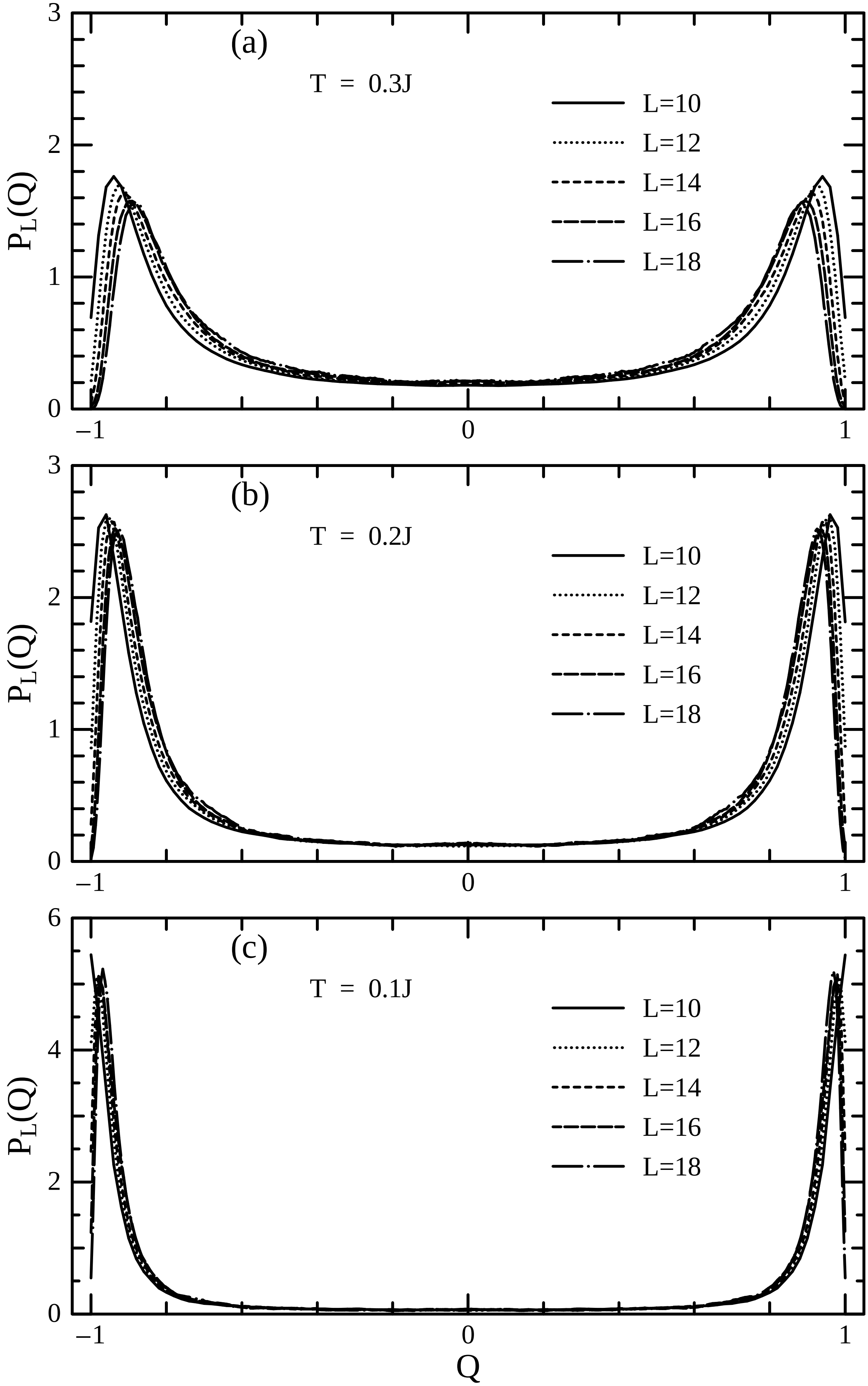
FIG. 1. $P_L(Q)$ versus Q at (a) $T = 0.3J$, (b) $T = 0.2J$ and (c) $T = 0.1J$ in the random Ising model with $a = 0.8$ on the square lattice of $L \times L$.

FIG. 2. Temperature dependences of g_L of the random Ising model with $a = 0.8$.

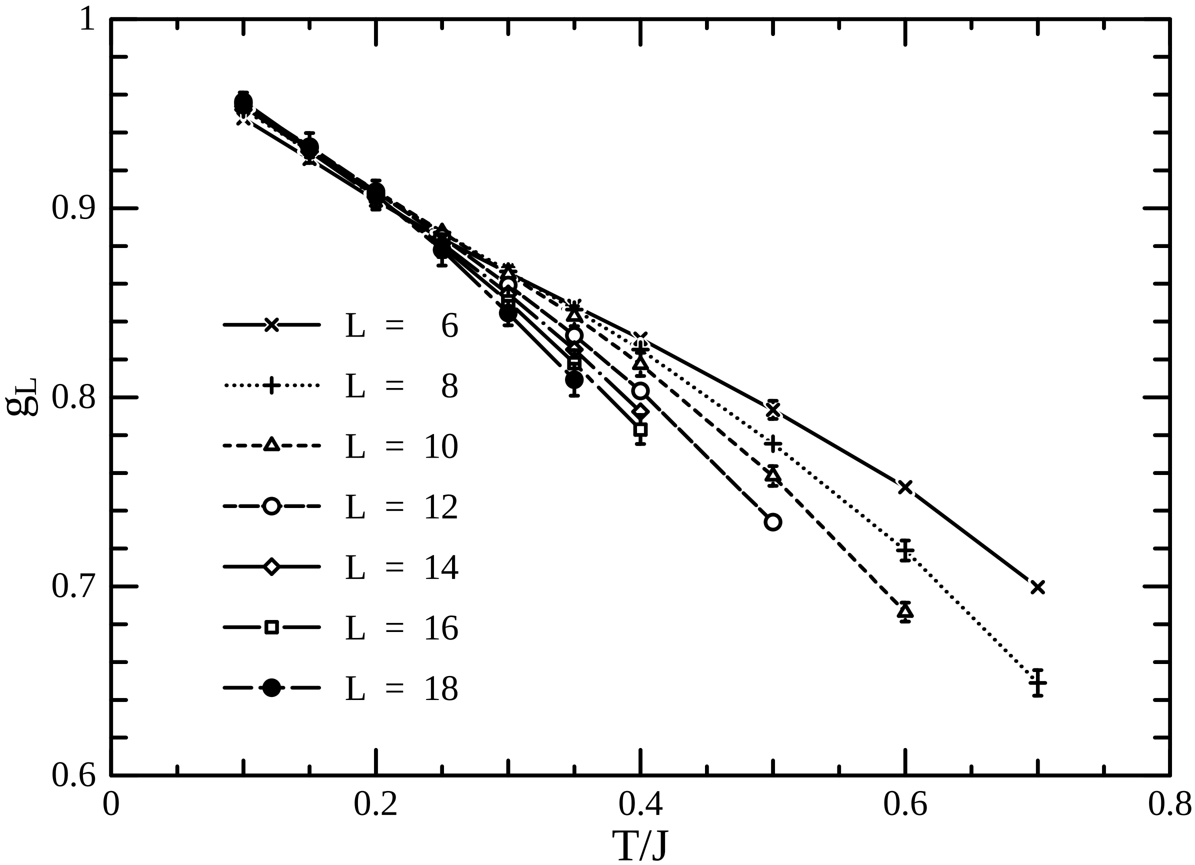
FIG. 3. Scaling plots of (a) g_L and (b) χ_{SG}, assuming $T_c = 0.19J$ and $\epsilon = (T - T_c)/J$.

FIG. 4. Scaling plots of (a) g_L and (b) χ_{SG}, assuming $T_c = 0$ and $\epsilon = (T - T_c)/J$.

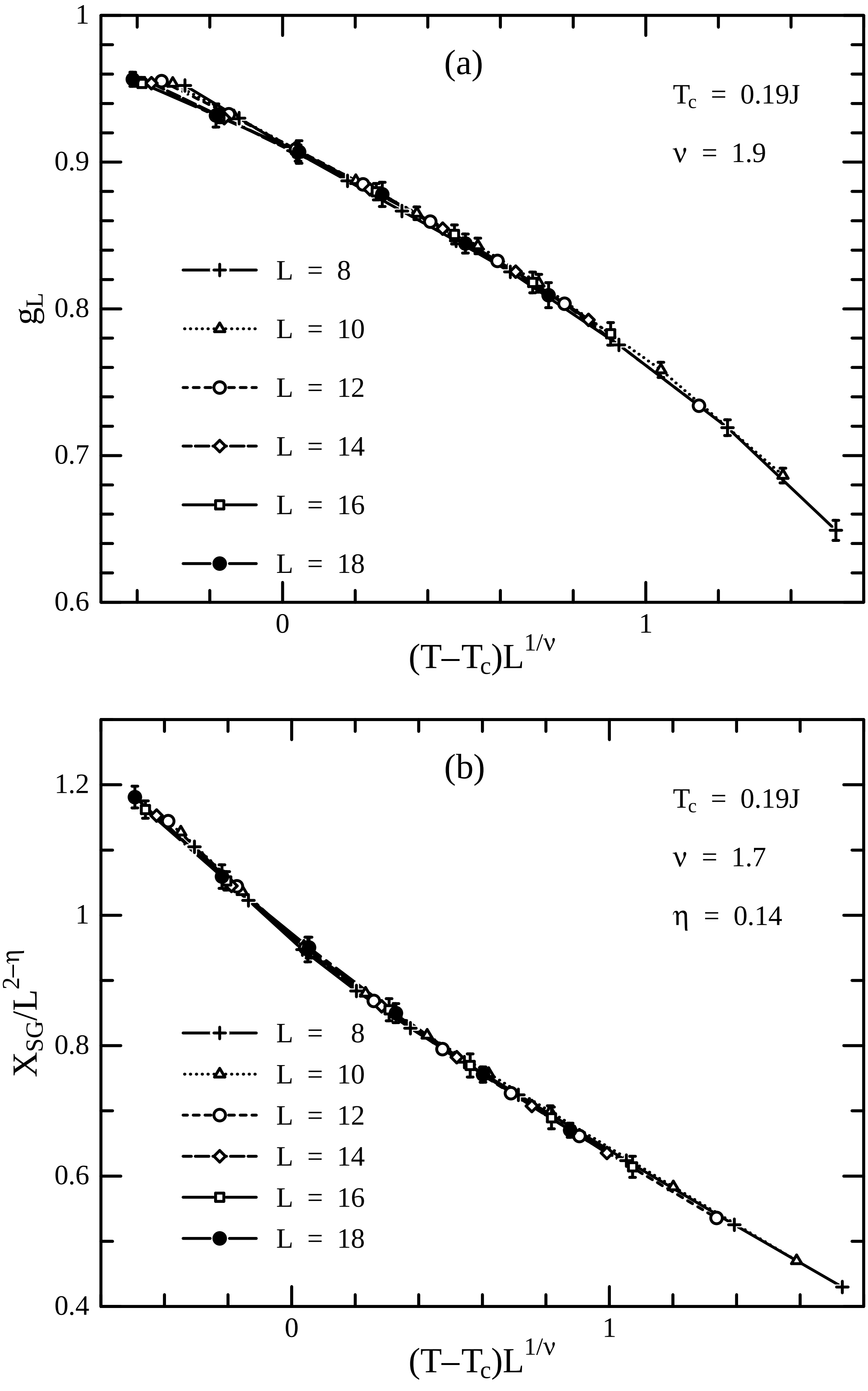
FIG. 5. Scaling plots of $P_L(Q)$ at (a) $T = 0.2J$ and (b) $T = 0.1J$.

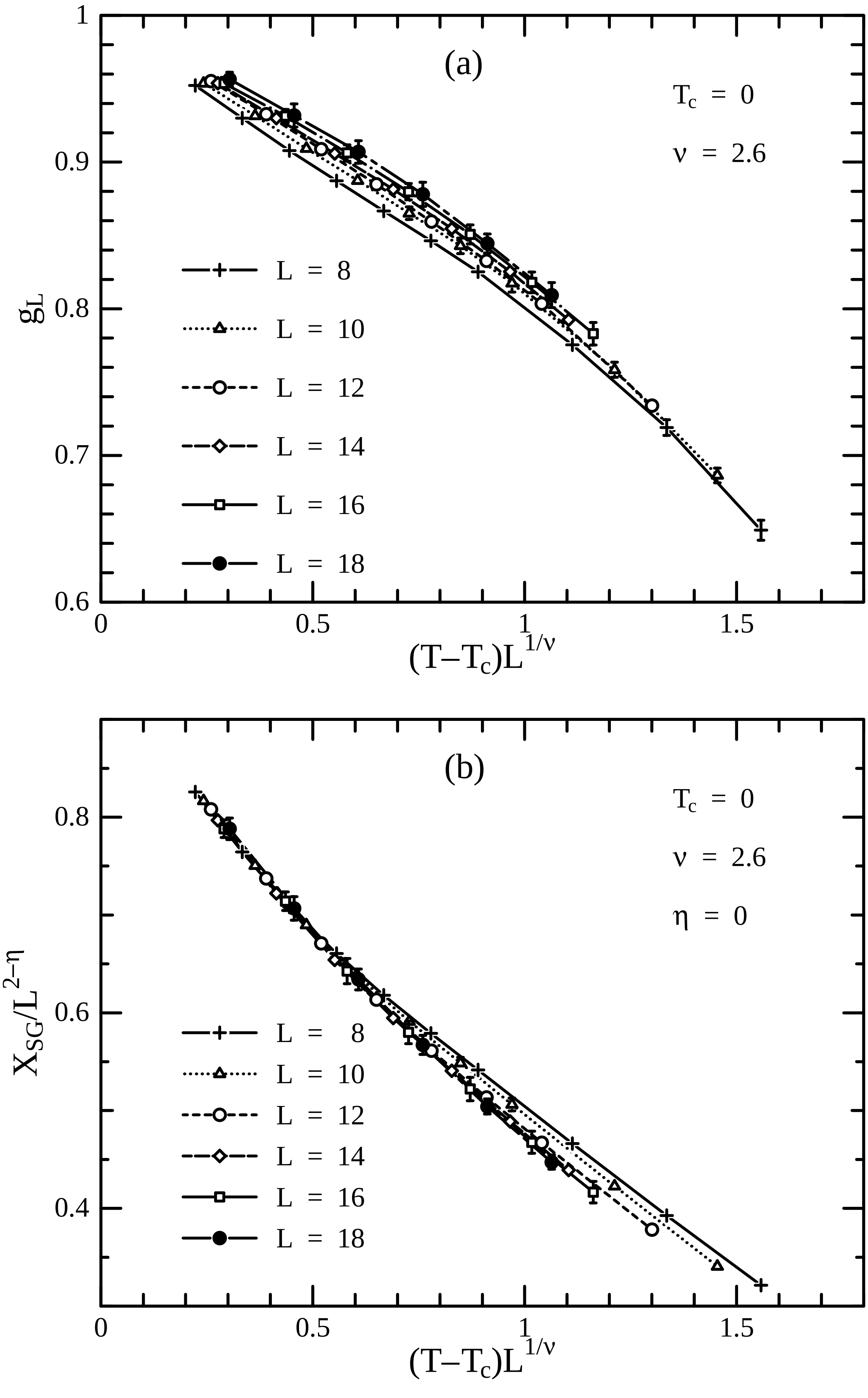


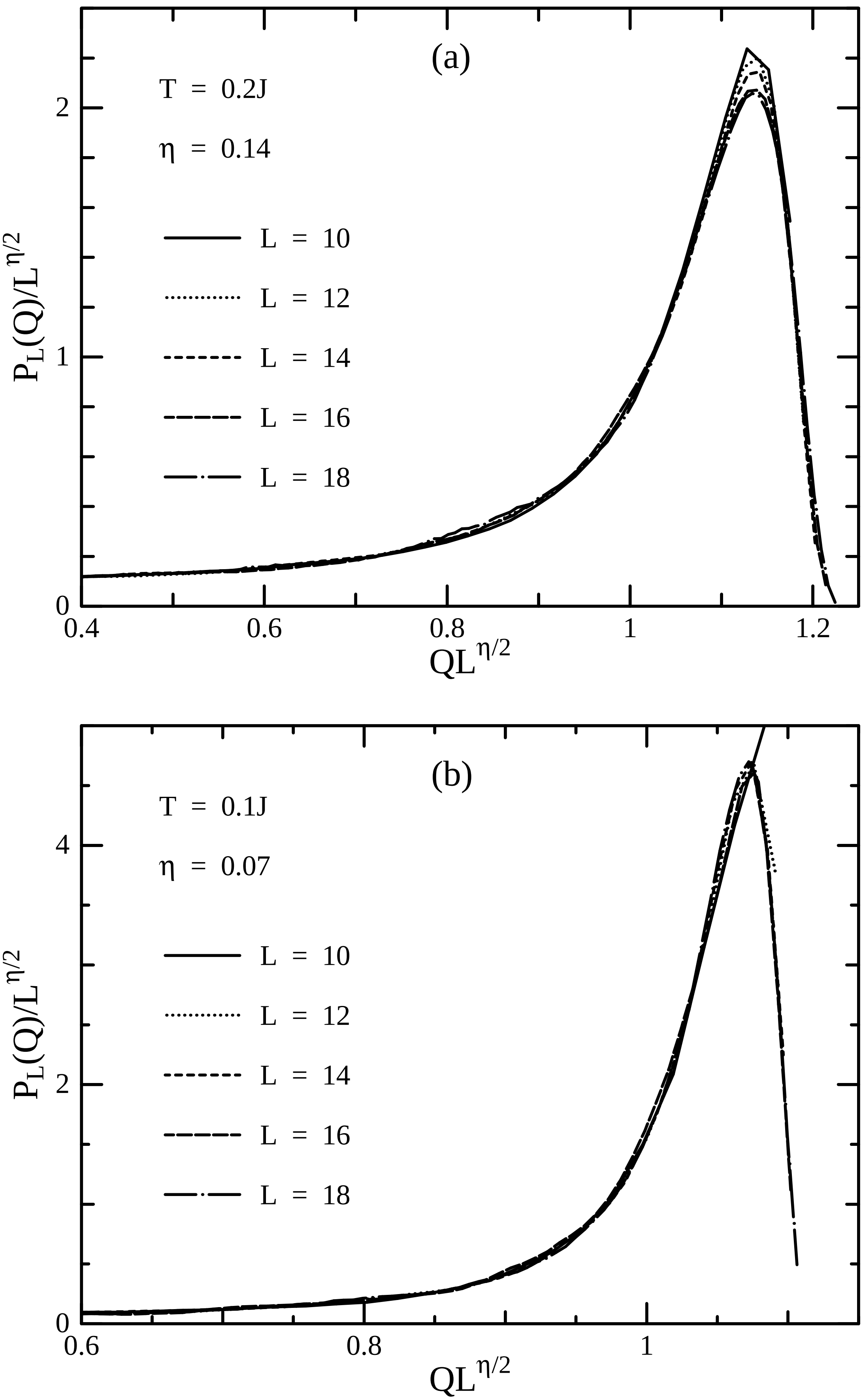
(a)
T = 0.3J
(b)
T = 0.2J
(c)
T = 0.1J
L=10
L=12
L=14
L=16
L=18
$P_L(Q)$
Q
3
2
1
0
6
4
−1
1



1
0.9
0.8
0.7
0.6
0
0.2
0.4
0.6
0.8
T/J
g_L
L = 6
L = 8
L = 10
L = 12
L = 14
L = 16
L = 18


(a)
T_c = 0.19J
ν = 1.9
L = 8
L = 10
L = 12
L = 14
L = 16
L = 18
g_L
(T−T_c)L^{1/ν}
(b)
T_c = 0.19J
ν = 1.7
η = 0.14
L = 8
L = 10
L = 12
L = 14
L = 16
L = 18
X_SG/L^{2−η}
(T−T_c)L^{1/ν}


(a)
T_c = 0
ν = 2.6
L = 8
L = 10
L = 12
L = 14
L = 16
L = 18
g_L
(T−T_c)L^{1/ν}
(b)
T_c = 0
ν = 2.6
η = 0
L = 8
L = 10
L = 12
L = 14
L = 16
L = 18
X_SG/L^{2−η}
(T−T_c)L^{1/ν}


(a)
T = 0.2J
η = 0.14
L = 10
L = 12
L = 14
L = 16
L = 18
$P_L(Q)/L^{\eta/2}$
$QL^{\eta/2}$
0
1
2
0.4
0.6
0.8
1
1.2
(b)
T = 0.1J
η = 0.07
L = 10
L = 12
L = 14
L = 16
L = 18
$P_L(Q)/L^{\eta/2}$
$QL^{\eta/2}$
0
2
4
0.6
0.8
1